hep-th/9607021 MIT-CTP-2543

Supersymmetry Breaking Through Confining and Dual Theory Gauge Dynamics [1]

Csaba Csáki, Lisa Randall,[2] Witold Skiba

Center for Theoretical Physics
Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

and

Robert G. Leigh

Department of Physics
Rutgers University
Piscataway, NJ 08855-0849, USA

Abstract

We show that theories in the confining, free magnetic, and conformal phases can break supersymmetry through dynamical effects. To illustrate this, we present theories based on the gauge groups $SU(n) \times SU(4) \times U(1)$ and $SU(n) \times SU(5) \times U(1)$ with the field content obtained by decomposing an $SU(m)$ theory with an antisymmetric tensor and $m-4$ antifundamentals.

[1]Supported in part by DOE under cooperative agreement #DE-FC02-94ER40818 and #DE-FG05-90ER40599.

[2]NSF Young Investigator Award, Alfred P. Sloan Foundation Fellowship, DOE Outstanding Junior Investigator Award.

Recently, there has been a dramatic increase in the number of dynamical supersymmetry breaking models [1, 2, 3, 4, 5]. In part this increase has been due to new methods for analyzing supersymmetric theories [6, 7]. While many of the first models for breaking supersymmetry had instanton or gaugino generated terms which kept fields away from the origin [1, 2, 8], recent work has argued that models in other phases can also break supersymmetry. In Ref. [4], it was argued that supersymmetry can be broken due to confinement. A nontrivial modification of the Kähler potential near the origin removes the supersymmetry preserving minimum. Alternatively, models with a quantum modified moduli space can also break supersymmetry [3] because the supersymmetry preserving origin is removed by the quantum modified constraint. Models in the conformal or free magnetic phase can also break supersymmetry. In the models which have been studied to now, these models broke supersymmetry through an O'Raifeartaigh mechanism in the dual theory [9, 10] or strong dynamics in the electric theory. A class of models described below is distinguished by the fact that the dynamics can be understood only in the dual description where dynamical effects are responsible for supersymmetry breaking.

In a recent paper [5], a new class of models was studied which were based on a product group in which supersymmetry is broken dynamically. There it was argued that supersymmetry breaking could be understood as a collusion between separate dynamical effects from the two nonabelian gauge groups. In the first example, the 4-3-1 model based on the gauge group $SU(4) \times SU(3) \times U(1)$, the exact superpotential could be found and the model was an O'Raifeartaigh model with both groups contributing to the final form of the superpotential. In all cases, supersymmetry breaking could be understood by taking a limit in which the gauge coupling of a confining gauge group is the biggest coupling. In this limit, Yukawa couplings which were necessary to lift flat directions turn into mass terms. Many flavors can be integrated out and the gauge dynamics of the second nonabelian gauge factor generated a superpotential which drives fields from the origin leading to the breaking of supersymmetry.

In the particular models considered in Ref. [5], other mechanisms of supersymmetry breaking could appear as well in the limit that one of the gauge couplings dominated. For example, in the particular case of the 4-3-1 model supersymmetry breaking occurs in the strong Λ_3 limit through confinement, analogous to the mechanism of Ref. [4]. On the other hand, if some of

the tree level terms are removed, supersymmetry breaking appears due to a quantum modified constraint [3]. Because of these additional descriptions, it was not clear that the quantum modified constraint was not essential to supersymmetry breaking.

In this paper, we show that analogous models in which each of the two groups is in one of a confining, free magnetic, or conformal phase (in the limit that we neglect the other coupling) also break supersymmetry, through a conspiracy of dynamical effects from the two gauge groups. Naively, it would appear that such models should allow fields to go to the origin. However, because of the tree-level superpotential and dynamics of one group, the other group can generate a dynamical superpotential in the infrared which forbids the origin and yields supersymmetry breaking.

It is interesting that models in which the theory must be analyzed at low energy in the dual phase can break supersymmetry. It is not essential for the number of flavors to be so small that a dynamical superpotential, a quantum modified constraint, or even confinement occurs in the electric theory. This suggests the possibility of a much larger class of supersymmetry breaking models because of the much less restrictive condition on the size of the initial particle content.

The two models we present in this paper are obvious generalizations of the models considered in Ref. [5]. Analogously to the n-3-1 models, supersymmetry breaking can be understood as a result of Yukawa couplings and strong dynamics which make flavors of the second gauge group heavy. In the resulting theory, the origin is forbidden because of a dynamical superpotential from the second gauge group. The mechanism is in some sense independent of the number of flavors in the initial theory. We present two classes of models to illustrate this. In the first class of models, in which one of the gauge groups is confining, supersymmetry breaking occurs through a conspiracy of gauge effects. We then consider a model which must be analyzed in the dual phase. The supersymmetry breaking dynamics for this model is remarkably similar to that of the confining theory, as we will show below.

The fields of the first model can be obtained by decomposing $SU(n+4)$ model with an antisymmetric tensor [8] into its $SU(n) \times SU(4) \times U(1)$ subgroup. The field content is

$$\begin{array}{c}\square\\[-4pt]\square\end{array} \;\rightarrow\; A(\begin{array}{c}\square\\[-4pt]\square\end{array},1)_8 + a(1,\begin{array}{c}\square\\[-4pt]\square\end{array})_{-2n} + T(\square,\square)_{4-n}$$

$$n \cdot \bar{\Box} \quad \rightarrow \quad \bar{F}_I(\bar{\Box}, 1)_{-4} + \bar{Q}_i(1, \bar{\Box})_n, \tag{1}$$

where $i, I = 1, \ldots, n$. We take the tree-level superpotential to be

$$\begin{aligned} W_{tree} &= A\bar{F}_1\bar{F}_2 + A\bar{F}_3\bar{F}_4 + \ldots + A\bar{F}_{n-2}\bar{F}_{n-1} \\ &+ a\bar{Q}_2\bar{Q}_3 + a\bar{Q}_4\bar{Q}_5 + \ldots + a\bar{Q}_{n-1}\bar{Q}_1 + T\bar{F}_1\bar{Q}_1 + \ldots + T\bar{F}_n\bar{Q}_n. \end{aligned} \tag{2}$$

A detailed analysis along the lines of Ref. [5] shows that this superpotential lifts all flat directions. The relative shift of the indices in the $A\bar{F}\bar{F}$ and $a\bar{Q}\bar{Q}$ terms is important. Without this shift not all flat directions are lifted. This superpotential preserves an R-symmetry which is anomalous only under the $U(1)$ gauge group.

We analyze this theory in the limit where $\Lambda_n \gg \Lambda_4$. The $SU(n)$ field content is an antisymmetric tensor, four fundamentals and n antifundamentals which give confining gauge dynamics. Below Λ_n, the effective degrees of freedom are the $SU(n)$ invariants [9]

$$\begin{aligned} & X_{IJ} = A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J} \\ & \bar{B} = \bar{F}_{\alpha_1 1}\ldots\bar{F}_{\alpha_n n}\epsilon^{\alpha_1\ldots\alpha_n} \\ & (B_1)^a = T^{\alpha_1 a}A^{\alpha_2\alpha_3}\ldots A^{\alpha_{n-1}\alpha_n}\epsilon_{\alpha_1\ldots\alpha_n} \\ & (B_3)_a = \epsilon_{abcd}T^{\alpha_1 b}T^{\alpha_2 c}T^{\alpha_3 d}A^{\alpha_4\alpha_5}\ldots A^{\alpha_{n-1}\alpha_n}\epsilon_{\alpha_1\ldots\alpha_n} \\ & M_I^a = T^{\alpha a}\bar{F}_{\alpha I}, \end{aligned} \tag{3}$$

plus the $SU(n)$ singlets a and $\bar{Q}_i$.

The superpotential is the sum of the tree-level terms from Eq. (2) and the confining superpotential [9].

$$\begin{aligned} W \;=\; & X_{12} + \ldots + X_{n-2,n-1} + a\bar{Q}_2\bar{Q}_3 + \ldots + a\bar{Q}_{n-1}\bar{Q}_1 + \\ & M_1\bar{Q}_1 + \ldots + M_n\bar{Q}_n + \frac{1}{\Lambda_n^{2n-1}}\Big(B_{3a}M_{I_1}^a X_{I_2I_3}\ldots X_{I_{n-1}I_n}\epsilon^{I_1\ldots I_n} \\ & + B_1^a M_{I_1}^b M_{I_2}^c M_{I_3}^d X_{I_4I_5}\ldots X_{I_{n-1}I_n}\epsilon^{I_1\ldots I_n}\epsilon_{abcd} + \bar{B}B_1^a B_{3a}\Big), \end{aligned} \tag{4}$$

where small Latin letters denote $SU(4)$ indices.

Note that in the confined theory, some of the Yukawa couplings have become mass terms. To deduce the infrared theory, we integrate out all massive fields. It is technically difficult to integrate out the fields using the full superpotential from Eq. (4). For simplicity we set the couplings of all

$a\bar{Q}\bar{Q}$ terms to zero. We will argue based on symmetries that the models with the additional baryon operators included still break supersymmetry. It should be noted that the flat directions now present classically are lifted in the quantum theory [3], which is presumably a valid supersymmetry breaking model as well.

Because we have integrated out n massive flavors, the $SU(4)$ theory at low energy has an antisymmetric tensor and only one flavor. This theory dynamically generates a superpotential. The low-energy superpotential is therefore

$$W_{\text{eff}} = X_{12} + \ldots + X_{n-2,n-1} + \frac{1}{\Lambda_n^{2n-1}}\bar{B}m + \left[\frac{\tilde{\Lambda}_4^5}{\text{Pf}a\, m}\right]^{\frac{1}{2}}, \tag{5}$$

where $\text{Pf}a = a^{ab}a^{cd}\epsilon_{abcd}$, $m = B_1^a B_{3a}$, and $\tilde{\Lambda}_4$ is the dynamical scale of the effective one flavor $SU(4)$ theory. The equations of motion have set most terms to zero in the Λ_n dependent term. The $\bar{B}$ equation of motion would set $m = 0$. However, this is inconsistent with the $\left[\frac{\tilde{\Lambda}_4^5}{\text{Pf}a\, m}\right]^{\frac{1}{2}}$ term in the superpotential, which drives m from the origin in a theory with no flat directions. Therefore, we conclude that the equations of motion are contradictory, and supersymmetry is dynamically broken.

We have argued that supersymmetry is broken in the theory with $\gamma^{ij} = 0$, where γ^{ij} is the coefficient of the $a\bar{Q}\bar{Q}$ operators in the tree-level superpotential. It is clear that even with nonzero γ^{ij}, supersymmetry is still broken. From symmetries, it can be shown that the neglected terms can correct the superpotential by a power series in

$$\mathcal{A} = \Lambda_n^{-2n+1}(\text{Pf}a)^{\frac{1}{2}}(X_{IJ})^{\frac{n-2}{2}} m^{\frac{1}{2}}(\gamma^{ij})(m^{iI})^{-2}, \tag{6}$$

where m^{iI} is the coefficient of the $T\bar{F}_I\bar{Q}_i$ operators. For small γ, these terms could only give a sufficiently large contribution to cancel a nonzero F-term at field values larger than Λ_n. In this case, the theory should have been analyzed in the Higgs phase, which is clearly inconsistent with supersymmetry since there were no flat directions.

As an aside, we note that in the version of the theory without the $a\bar{Q}\bar{Q}$ terms in the superpotential (and hence without the corrections of Eq. (6)), there is an additional source of supersymmetry breaking. The terms $X_{12} + \ldots + X_{n-2,n-1}$ in the superpotential lead to supersymmetry breaking due to

confinement, as described in Ref. [4]. Here we emphasize the first argument for supersymmetry breaking, which generalizes beyond confining models, as we describe below.

Next, we consider theories based on the gauge group $SU(n)\times SU(5)\times U(1)$ (n even) obtained by reducing the gauge group of the $SU(n+5)$ theory with an antisymmetric tensor and $n+1$ antifundamentals. The mechanism of supersymmetry breaking will turn out to be very similar to the previous models, despite the very different gauge dynamics.

The field content is

$$\begin{aligned} \begin{smallmatrix}\square\\\square\end{smallmatrix} \quad &\rightarrow \quad A(\begin{smallmatrix}\square\\\square\end{smallmatrix},1)_{10} + a(1,\begin{smallmatrix}\square\\\square\end{smallmatrix})_{-2n} + T(\square,\square)_{5-n} \\ (n+1)\cdot\overline{\square} \quad &\rightarrow \quad \bar{F}_I(\overline{\square},1)_{-5} + \bar{Q}_i(1,\overline{\square})_n, \end{aligned} \tag{7}$$

where $i, I = 1,\ldots,n+1$. The tree-level superpotential is

$$\begin{aligned} W_{tree} \;=\;& A\bar{F}_1\bar{F}_2 + \ldots + A\bar{F}_{n-1}\bar{F}_n + a\bar{Q}_2\bar{Q}_3 + \ldots + a\bar{Q}_n\bar{Q}_1 + \\ & T\bar{F}_1\bar{Q}_1 + \ldots + T\bar{F}_{n+1}\bar{Q}_{n+1}. \end{aligned} \tag{8}$$

Again a detailed analysis verifies the absence of flat directions.

The $SU(5)$ gauge group has an antisymmetric tensor and n flavors while the $SU(n)$ has an antisymmetric tensor and five flavors. The $SU(5)$ group is in the conformal regime while the $SU(n)$ group is in the free magnetic phase. Although it seems more obvious to dualize the $SU(n)$ which is in the free magnetic phase it is simpler to dualize the gauge group $SU(5)$, as it has an odd number of colors. This duality will increase the number of $SU(n)$ flavors by $n-3$ which takes the theory out of the free magnetic phase.

The dual description of $SU(5)$ with an antisymmetric tensor and n flavors is an $SU(n-3)\times Sp(2n-8)$ gauge theory[9] with the field content given in Table 1.

The $SU(n-3)\times Sp(2n-8)$ gauge group in Table 1 is the dual of the $SU(5)$ gauge group, while the $SU(n)\times U(1)$ is the remaining original gauge group unchanged by the duality transformation. The $SU(n+1)_{\bar{Q}}\times SU(n+1)_{\bar{F}}$ global symmetries are the non-abelian global symmetries of the original $SU(n)\times SU(5)\times U(1)$ theory.

The superpotential consists of the terms corresponding to the tree-level superpotential of Eq. (8) and the terms arising from the duality transformation. It is given by

$$W \;=\; A\bar{F}_1\bar{F}_2 + \ldots + A\bar{F}_{n-1}\bar{F}_n + H_{23} + \ldots + H_{n1} + M_1\bar{F}_1 + \ldots +$$

	$SU(n-3)$	$Sp(2n-8)$	$SU(n)$	$U(1)$	$SU(n+1)_{\bar{Q}}$	$SU(n+1)_{\bar{F}}$
A	1	1	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	10	1	1
$\bar{F}$	1	1	$\overline{\Box}$	-5	1	$\Box$
x	$\Box$	$\Box$	1	0	1	1
p	$\Box$	1	1	$5n$	1	1
$\bar{a}$	$\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	1	1	0	1	1
$\bar{q}$	$\overline{\Box}$	1	$\overline{\Box}$	-5	1	1
l	1	$\Box$	1	0	$\overline{\Box}$	1
M	1	1	$\Box$	5	$\Box$	1
H	1	1	1	0	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1
B_1	1	1	$\Box$	$5(1-n)$	1	1

(9)

Table 1: The field content of the $SU(n)\times SU(5)\times U(1)$ theory after dualizing the $SU(5)$ gauge group.

$$M_{n+1}\bar{F}_{n+1} + M\bar{q}lx + Hl^2 + B_1 p\bar{q} + \bar{a}x^2. \tag{10}$$

As in the $SU(n)\times SU(4)\times U(1)$ models, some of the tree-level Yukawa terms are mapped into mass terms in the dual description. To simplify the theory we again set the coefficients of the $A\bar{F}\bar{F}$ operators to zero, though in this case it is not difficult to leave them in. With this simplification, one can easily integrate out the massive flavors of $SU(n)$ since the $\bar{F}_I$ equations of motion set all M's to zero. There is just one $SU(n)$ flavor remaining and thus there is a dynamically generated term in the superpotential from the $SU(n)$ dynamics. The effective low-energy superpotential is

$$W = H_{23} + H_{45} + \ldots + H_{n1} + Hl^2 + \bar{a}x^2 + \tilde{M}p + \frac{\tilde{\Lambda}_n^{n+1}}{(\tilde{M}\tilde{X}^{(n-4)/2}\mathrm{Pf}\,A)^{1/2}}, \tag{11}$$

where $\tilde{M} = B_1\bar{q}$, $\tilde{X} = A\bar{q}\bar{q}$ and $\mathrm{Pf}\,A = A^{n/2}$, while $\tilde{\Lambda}_n$ is the effective $SU(n)$ scale. This superpotential looks very much like the one in Eq. (5), with $\tilde{M}$ playing the role of m and p the role of $\bar{B}$. The equations of motion are again contradictory. We again conclude that supersymmetry is broken.

The above analysis neglected the $Sp(2n-8)$ group that appears from dualizing the $SU(5)$ group. This group is however Higgsed by the VEV's of

the l fields as a result of the H equations of motion and the terms linear in H in the superpotential. Although instanton terms can be generated in the broken $Sp(2n-8)$ group, these will not involve the fields $\tilde{M}, \tilde{X}, \mathrm{Pf}A$ or p and therefore do not affect the proof of dynamical supersymmetry breaking given above. The $Sp(2n-8)$ dynamics seems to be irrelevant to the analysis of the model.

The dynamics of the general $SU(n) \times SU(m) \times U(1)$ models $(n, m \geq 5)$ obtained in the same way is very similar to that of the $SU(n) \times SU(5) \times U(1)$ model, if one dualizes the $SU(n)$ corresponding to odd n. We expect that a similarly constructed tree-level superpotential lifts all flat directions. One can then show that the resulting low-energy superpotential is in one-to-one correspondence to the superpotential of Eq. (11), with the remaining gauge group being $SU(m-3) \times Sp(2m-8) \times SU(m) \times U(1)$ (m is even), which is obtained by dualizing the original $SU(n)$ group. Since the superpotential is exactly of the same form as the one in Eq. (11) we conclude that the general $SU(n) \times SU(m) \times U(1)$ models break supersymmetry as well.

The similarities between the $SU(n) \times SU(4) \times U(1)$ and $SU(n) \times SU(5) \times U(1)$ models is intriguing. In both models, the dynamics of the $SU(n)$ group leads to additional flavors of the second gauge group, in one case due to confinement, and in the other case, due to the dual description. In both cases, some of the tree level terms are mapped into mass terms due to dynamical effects in the $SU(n)$ gauge group. After integrating out these massive flavors the other gauge group has only a single flavor remaining besides the antisymmetric tensor and produces a dynamically generated superpotential. This dynamical superpotential together with a piece of the superpotential from the strong dynamics of the first group breaks supersymmetry. Thus supersymmetry breaking in these theories involves a subtle interplay between the gauge dynamics of both groups and the tree-level superpotential.

That these theories (and presumably the general $SU(n) \times SU(m) \times U(1)$ models as well) break supersymmetry suggests the existence of still more models of dynamical supersymmetry breaking. The flavor content of these models can be much larger than one would naively have anticipated by the requirement of a dynamical superpotential, because Yukawa couplings or other interactions in the presence of strong dynamics can change the phase of the theory in the infrared. The low-energy description might then have sufficiently few flavors to break supersymmetry dynamically.

Acknowlegements: Conversations with Erich Poppitz and Riccardo Rattazzi are gratefully acknowledged. We note that, after this work was completed, a similar discussion of supersymmetry breaking appeared in Ref. [11] in the context of product group theories with fundamental matter.

References

[1] M. Dine, A.N. Nelson, Y. Nir and Y. Shirman, Phys. Rev. **D53** (1996) 2658.

[2] E. Poppitz and S.P. Trivedi, Phys. Lett. **B365** (1996) 125.

[3] K. Intriligator and S. Thomas, `hep-th/9603158`.

[4] K. Intriligator, N. Seiberg and S.H. Shenker, Phys. Lett. **B342** (1995) 152.

[5] C. Csáki, L. Randall and W. Skiba, `hep-th/9605108`.

[6] N. Seiberg, Phys. Rev. **D49** (1994) 6857;
K. Intriligator, R.G. Leigh and N. Seiberg, Phys. Rev. **D50** (1994) 1092;
K. Intriligator, Phys. Lett. **B336** (1994) 409.

[7] N. Seiberg, Nucl. Phys. **B435** (1995) 129.

[8] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B256** (1985) 557.

[9] P. Pouliot, Phys. Lett. **B367** (1996) 151.

[10] P. Pouliot and M. Strassler, Phys. Lett. **B370** (1996) 76.

[11] E. Poppitz, Y. Shadmi and S. P. Trivedi, `hep-th/9606184`.